

One-step generation of multi-atom Greenberger-Horne-Zeilinger states in separate cavities via adiabatic passage

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We propose a scheme to deterministically generate Greenberger-Horne-Zeilinger states of $N \geq 3$ atoms trapped in spatially separated cavities connected by optical fibers. The scheme is based on the technique of fractional stimulated Raman adiabatic passage which is one-step in the sense that one needs just wait for the desired entangled state to be generated in the stationary regime. The parametrized shapes of the Rabi frequencies of the classical fields that drive the two end atoms are chosen appropriately to realize the scheme. We also show numerically that the proposed scheme is insensitive to the fluctuations of the pulses' parameters and, at the same time, robust against decoherence caused by the dissipation due to fiber decay. Moreover, a relatively high fidelity can be obtained even in the presence of cavity decay and atomic spontaneous emission.

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I. INTRODUCTION

Entanglement is not only an essential ingredient for testing quantum nonlocality against local hidden theories [1, 2], but also a necessary resource for implementing various quantum informatic tasks. Fundamentally, it is one of the most important traits in quantum

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mechanics. It has found different applications in quantum information processing (QIP) such as quantum cryptography [3], quantum teleportation [4, 5], quantum dense coding [6, 7], quantum secret sharing [8], and so on. Typical entangled states are Bell states [1], Greenberger-Horne-Zeilinger (GHZ) states [2] and W states [9] have been identified and can directly be utilized in QIP [3–8]. In particular, great interest has been arisen regarding the significant role of GHZ states in the foundations of quantum mechanics measurement theory and quantum communication [10–14], error correction protocols [15, 16], and high-precision spectroscopy [17, 18]. Contrary to bipartite entangled states, GHZ states exhibit a special kind of entanglement between $N \geq 3$ parties, providing a possibility to test quantum non-locality in a one-shot manner. So, of common interest is the problem of how to generate GHZ states using current technologies. In fact, for trapped ions [20] and photons [21, 22], a series of experimental methods [19, 20, 23] have already been invented. It is worthy to note that cavity quantum electrodynamics (CQED) proves to be very promising for QIP. Based on CQED, numerous schemes [24–31] have been proposed for deterministic generation of entanglement between atoms trapped in different cavities connected by optical fibers [32–36]. For example, Zheng *et al.* proposed a simplified scheme to product GHZ states [37], while Li *et al.* made use of the quantum Zeno dynamics [38]. Both the schemes are difficult to implement because they depend on the exact knowledge of all parameters and require controlling the interaction time accurately. A way to overcome such difficulties in state engineering is to force the system’s initial state evolve along a dark-state, if any, by means of adiabatic passage. Such an evolution can be realized by the so-called technique of stimulated Raman adiabatic passage (STIRAP) [39] or fractional STIRAP (f-STIRAP) [40], which have been employed in the context of coherent population transfer [41, 42] and coherent atomic beam deflection [43].

In this paper, we design a one-step scheme to deterministically generate GHZ states of N atoms individually trapped in a linear array of optical cavities whose nearest neighbors are connected by $N - 1$ optical fibers. Two external lasers are needed to drive the two end atom-cavity subsystems. The scheme is based on the adiabatic passage along a dark state, which is a specific eigenstate of the total atom-cavity-fiber system corresponding to the zero eigenvalue. The key idea is to choose suitable time-dependent Rabi frequencies of the driving lasers. Here, we use Gaussian pulses with proper parameters which are turned on and turned off in an adequate manner so that in the long-time (stationary) limit the desired GHZ states

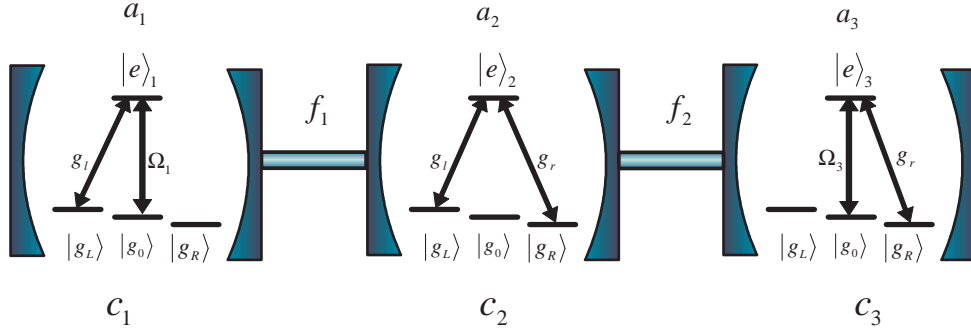


FIG. 1: Three atoms (a_1 , a_2 and a_3), each of which has one excited state ($|e\rangle_k$ ($k = 1, 2, 3$)) and three ground states ($|g_L\rangle_k$, $|g_0\rangle_k$ and $|g_R\rangle_k$) with a tripod-type configuration, are respectively trapped in three optical cavities (c_1 , c_2 and c_3) connected by two optical fibers (f_1 and f_2).

emerge automatically. Compared with the previous schemes [37, 38], ours has the following advantages: (i) The multi-atom GHZ states are produced deterministically only in one step without worrying about precise control over interaction time; (ii) The scheme is insensitive to moderate fluctuations of experimental parameters and (iii) The process is immune to the fiber decay, and a relatively high fidelity can be obtained even in the presence of cavity decay and atomic spontaneous emission.

Our paper is organized as follows. After the Introduction, in section II, we describe the physical model and present the detailed procedure to realize the scheme for generating three-atom GHZ states. The general case of any $N > 3$ atoms is also touched upon briefly in this section. Then, in section III, we discuss issues regarding the robustness of our scheme against possible fluctuations in the parameters involved as well as against various mechanisms of decoherence. Finally, we conclude in section IV.

II. GENERATION OF MULTI-ATOM GHZ STATES

We first consider the case of three atoms in detail. As shown in Fig. 1, three atoms a_1 , a_2 and a_3 are trapped in three distant linearly arranged optical cavities c_1 , c_2 and c_3 , respectively. Each atom has one excited state $|e\rangle_k$ ($k = 1, 2, 3$) and three ground states $|g_L\rangle_k$, $|g_0\rangle_k$ and $|g_R\rangle_k$, which are nondegenerate and correspond to $J = 1$ and $m = -1, 0, +1$, respectively. The cavities c_1 and c_3 are single-mode while the cavity c_2 is two-mode. They are connected by two short optical fibers f_1 and f_2 . The atomic transitions $|e\rangle_{1(2)} \leftrightarrow |g_L\rangle_{1(2)}$

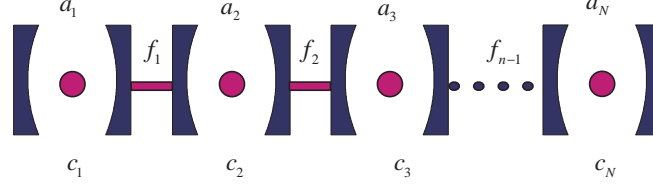


FIG. 2: N atoms are respectively trapped in N cavities through $N - 1$ fibers.

$(|e\rangle_{2(3)} \leftrightarrow |g_R\rangle_{2(3)})$ are resonantly coupled to the left-circularly (right-circularly) polarized cavity mode, while the transitions $|e\rangle_1 \leftrightarrow |g_0\rangle_1$ ($|e\rangle_3 \leftrightarrow |g_0\rangle_3$) is driven resonantly by an external classical laser.

In the short-fiber limit, $L\nu/(2\pi c) \ll 1$ [29, 44] with L the fiber length, c the speed of light and ν the decay rate of the cavity field into a continuum of fiber modes, only one resonant fiber mode interacts with the cavity mode. Then, in the interaction picture, the Hamiltonian of the total atom-cavity-fiber system can be written as ($\hbar = 1$)

$$H = H_{al} + H_{ac} + H_{cf}, \quad (1)$$

$$H_{al} = \Omega_1(t)e^{i\varphi_1}|e\rangle_1\langle g_0| + \Omega_3(t)e^{i\varphi_3}|e\rangle_3\langle g_0| + H.c., \quad (2)$$

$$H_{ac} = \sum_{i=1}^2 g_{i,l}a_{i,l}|e\rangle_i\langle g_L| + \sum_{i=2}^3 g_{i,r}a_{i,r}|e\rangle_i\langle g_R| + H.c., \quad (3)$$

$$H_{cf} = v_1b_1^\dagger(a_{1,l} + a_{2,l}) + v_2b_2^\dagger(a_{2,r} + a_{3,r}) + H.c., \quad (4)$$

with $\Omega_{1(3)}(t)$ and $\varphi_{1(3)}$ the Rabi frequencies and phases of the driving lasers, $a_{i,l(r)}^\dagger$ and $a_{i,l(r)}$ the creation and annihilation operators for the left-circularly (right-circularly) polarized mode of cavity c_i , and b_f^\dagger (b_f) the creation (annihilation) operators of the resonant mode of fiber f_f . For simplicity, we assume equal atom-cavity and equal cavity-fiber coupling strengths, i.e., $g_{i,l} = g_{i,r} = g$ and $v_1 = v_2 = v$. We also denote by $|n\rangle_{c1(c3)}$ the quantum field state of cavity c_1 (c_3) containing n left-circularly (right-circularly) polarized photons, by $|m, n\rangle_{c2}$ the quantum field state of cavity c_2 containing m left-circularly and n right-circularly polarized photons, and by $|n\rangle_{f1(f2)}$ the quantum field state of fiber f_1 (f_2) containing n photons.

Let the total system be initially in the separable state

$$|\Psi(-\infty)\rangle = |g_0, g_L, g_R\rangle_{a_1a_2a_3}|0\rangle_{c1}|0\rangle_{f1}|0, 0\rangle_{c2}|0\rangle_{f2}|0\rangle_{c3}. \quad (5)$$

Then, governed by the Hamiltonian in Eq. (1), it will evolve in a closed subspace spanned by 11 basis states $\{|\phi_l\rangle; l = 1, 2, \dots, 11\}$:

$$|\phi_1\rangle = |\Psi(-\infty)\rangle = |g_0, g_L, g_R\rangle_{a_1 a_2 a_3} |0\rangle_{c1} |0\rangle_{f1} |0, 0\rangle_{c2} |0\rangle_{f2} |0\rangle_{c3}, \quad (6)$$

$$|\phi_2\rangle = |e, g_L, g_R\rangle_{a_1 a_2 a_3} |0\rangle_{c1} |0\rangle_{f1} |0, 0\rangle_{c2} |0\rangle_{f2} |0\rangle_{c3}, \quad (7)$$

$$|\phi_3\rangle = |g_L, g_L, g_R\rangle_{a_1 a_2 a_3} |1\rangle_{c1} |0\rangle_{f1} |0, 0\rangle_{c2} |0\rangle_{f2} |0\rangle_{c3}, \quad (8)$$

$$|\phi_4\rangle = |g_L, g_L, g_R\rangle_{a_1 a_2 a_3} |0\rangle_{c1} |1\rangle_{f1} |0, 0\rangle_{c2} |0\rangle_{f2} |0\rangle_{c3}, \quad (9)$$

$$|\phi_5\rangle = |g_L, g_L, g_R\rangle_{a_1 a_2 a_3} |0\rangle_{c1} |0\rangle_{f1} |1, 0\rangle_{c2} |0\rangle_{f2} |0\rangle_{c3}, \quad (10)$$

$$|\phi_6\rangle = |g_L, e, g_R\rangle_{a_1 a_2 a_3} |0\rangle_{c1} |0\rangle_{f1} |0, 0\rangle_{c2} |0\rangle_{f2} |0\rangle_{c3}, \quad (11)$$

$$|\phi_7\rangle = |g_L, g_R, g_R\rangle_{a_1 a_2 a_3} |0\rangle_{c1} |0\rangle_{f1} |0, 1\rangle_{c2} |0\rangle_{f2} |0\rangle_{c3}, \quad (12)$$

$$|\phi_8\rangle = |g_L, g_R, g_R\rangle_{a_1 a_2 a_3} |0\rangle_{c1} |0\rangle_{f1} |0, 0\rangle_{c2} |1\rangle_{f2} |0\rangle_{c3}, \quad (13)$$

$$|\phi_9\rangle = |g_L, g_R, g_R\rangle_{a_1 a_2 a_3} |0\rangle_{c1} |0\rangle_{f1} |0, 0\rangle_{c2} |0\rangle_{f2} |1\rangle_{c3}, \quad (14)$$

$$|\phi_{10}\rangle = |g_L, g_R, e\rangle_{a_1 a_2 a_3} |0\rangle_{c1} |0\rangle_{f1} |0, 0\rangle_{c2} |0\rangle_{f2} |0\rangle_{c3}, \quad (15)$$

and

$$|\phi_{11}\rangle = |g_L, g_R, g_0\rangle_{a_1 a_2 a_3} |0\rangle_{c1} |0\rangle_{f1} |0, 0\rangle_{c2} |0\rangle_{f2} |0\rangle_{c3}. \quad (16)$$

The Hamiltonian (1), in terms of the basis states Eqs. (6 - 16), reads

$$H = \begin{pmatrix} 0 & \tilde{\Omega}_1(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \tilde{\Omega}_1^*(t) & 0 & g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & g & 0 & v & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & v & 0 & v & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & v & 0 & g & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g & 0 & g & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & g & 0 & v & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & v & 0 & v & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & v & 0 & g & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & g & 0 & \tilde{\Omega}_3(t) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{\Omega}_3^*(t) & 0 & 0 \end{pmatrix}, \quad (17)$$

with $\tilde{\Omega}_{1(3)}(t) \equiv \Omega_{1(3)}(t)e^{-i\varphi_{1(3)}}$.

An alternative basis of the subspace consists of 11 time-dependent eigenstates of the instantaneous Hamiltonian given by Eq. (17) $\{|\Phi_m(t)\rangle, m = 1, 2, \dots, 11\}$. It can be verified that one of the eigenvalues of such H is zero and the corresponding eigenstate, which we label by $|\Phi_1(t)\rangle$, has the form

$$\begin{aligned} |\Phi_1(t)\rangle = & \frac{G(t)}{\sqrt{4X^2(t) + G^2(t)[X^2(t) + 1]}} |\phi_1\rangle \\ & - \frac{X(t)}{\sqrt{4X^2(t) + G^2(t)[X^2(t) + 1]}} (|\phi_3\rangle - |\phi_5\rangle + |\phi_7\rangle - |\phi_9\rangle) \\ & - \frac{e^{i(\varphi_1 + \varphi_3)} G(t) X(t)}{\sqrt{4X^2(t) + G^2(t)[X^2(t) + 1]}} |\phi_{11}\rangle, \end{aligned} \quad (18)$$

where $X(t) = \Omega_1(t)/\Omega_3(t)$ and $G(t) = g/\Omega_3(t)$. The state $|\Phi_1(t)\rangle$ is called a trapped or dark state since it contains atoms only in the ground states during the entire evolution. The fact that excited levels $|e\rangle_k$ are missing in $|\Phi_1(t)\rangle$ is due to the destructive quantum interference. Namely, as seen from Eqs. (6 - 16), $|\phi_2\rangle$ ($|\phi_6\rangle, |\phi_{10}\rangle$) does contain the excited level $|e\rangle_1$ ($|e\rangle_2, |e\rangle_3$), but, the transition $|\phi_1\rangle \rightarrow |\phi_2\rangle$ ($|\phi_5\rangle \rightarrow |\phi_6\rangle, |\phi_9\rangle \rightarrow |\phi_{10}\rangle$) is canceled by the transition $|\phi_3\rangle \rightarrow |\phi_2\rangle$ ($|\phi_7\rangle \rightarrow |\phi_6\rangle, |\phi_{11}\rangle \rightarrow |\phi_{10}\rangle$). It is also of a surprise that the fibers' modes b_1 and b_2 do not appear in $|\Phi_1(t)\rangle$ (i.e., $|\phi_4\rangle$ and $|\phi_8\rangle$ that contain a fiber mode are absent in $|\Phi_1(t)\rangle$). This is again a consequence of destructive quantum interference: the transitions $|\phi_3\rangle \rightarrow |\phi_4\rangle$ ($|\phi_7\rangle \rightarrow |\phi_8\rangle$) and $|\phi_5\rangle \rightarrow |\phi_4\rangle$ ($|\phi_9\rangle \rightarrow |\phi_8\rangle$) destroy each other completely. However, both the atomic excited levels and the fibers' modes are important, via their virtual excitations, in connecting the different atomic ground levels as well as the different cavities, a very necessary feature in our system to generate entanglement between separated atoms.

The total atom-cavity-fiber system state $|\Psi(t)\rangle$ at any time t can be extended as a superposition of $\{|\Phi_m(t)\rangle\}$,

$$|\Psi(t)\rangle = \sum_{m=1}^{11} w_m(t) |\Phi_m(t)\rangle, \quad \sum_{m=1}^{11} |w_m(t)|^2 = 1, \quad (19)$$

with $|w_m(t)|^2$ the probability of finding the system in state $|\Phi_m(t)\rangle$. If the pulse that drives the atom a_3 precedes the pulse that drives the atom a_1 , i.e.,

$$\lim_{t \rightarrow -\infty} \Omega_3(t) > \lim_{t \rightarrow -\infty} \Omega_1(t) = 0, \quad (20)$$

or $\lim_{t \rightarrow -\infty} X(t) = X(-\infty) = 0$, then from Eqs. (18), (19) and (20) it follows that $|\Phi_1(-\infty)\rangle = |\phi_1\rangle = |\Psi(-\infty)\rangle$. Using this in Eq. (19) yields $w_{m=1}(-\infty) = 1$ and

$w_{m>1}(-\infty) = 0$. If we vary the pulses slowly enough in time to satisfy the adiabatic following condition, then the system initial state should evolve only along $|\Phi_1(t)\rangle$ of Eq. (18). The key strategy is to tailor the pulses so that

$$\frac{2\Omega_1(t)\Omega_3(t)}{\sqrt{\Omega_1^2(t) + \Omega_3^2(t)}} \ll g, \quad (21)$$

(or $2X(t)/\sqrt{X^2(t) + 1} \ll G(t)$) to keep the probabilities of finding the system in states $\{|\phi_3\rangle, |\phi_5\rangle, |\phi_7\rangle, |\phi_9\rangle\}$ negligible during the evolution and in the long-time limit both the pulses vanish simultaneously retaining their ratio constant, i.e.,

$$\lim_{t \rightarrow \infty} \Omega_{1,3}(t) = 0; \quad \lim_{t \rightarrow \infty} [\Omega_1(t)/\Omega_3(t)] = \lim_{t \rightarrow \infty} X(t) = X(\infty) = \text{const}. \quad (22)$$

If so

$$\lim_{t \rightarrow \infty} |\Phi_1(t)\rangle = |ghz\rangle_{123} \otimes |0\rangle_{c1}|0\rangle_{f1}|0,0\rangle_{c2}|0\rangle_{f2}|0\rangle_{c3}, \quad (23)$$

with

$$|ghz\rangle_{123} = \frac{1}{\sqrt{X^2(\infty) + 1}} |g_0, g_L, g_R\rangle_{a_1 a_2 a_3} - e^{i(\varphi_1 + \varphi_3)} \frac{X(\infty)}{\sqrt{X^2(\infty) + 1}} |g_L, g_R, g_0\rangle_{a_1 a_2 a_3}, \quad (24)$$

which for $X(\infty) = 1$ and $\varphi_1 + \varphi_3 = \pi$ is the desired GHZ state $|GHZ\rangle_{123} = (|g_0, g_L, g_R\rangle + |g_L, g_R, g_0\rangle)_{a_1 a_2 a_3} / \sqrt{2}$.

We next briefly present the generalization of the above scheme to the case of $N > 3$ atoms. As Fig. 2 shows, the N atoms a_1, a_2, \dots, a_N are respectively trapped in N cavities c_1, c_2, \dots, c_N connected by $N - 1$ fibers f_1, f_2, \dots, f_{N-1} . The level configurations of atoms $a_1, \{a_2, a_3, \dots, a_{N-1}\}$ and a_N are the same as those of atoms a_1, a_2 and a_3 in the case of $N = 3$. For example, for an odd $N > 3$ and equal atom-cavity and equal cavity-fiber coupling strengths, the expressions of H_{al} , H_{ac} and H_{cf} in the total Hamiltonian H read

$$H_{al} = \Omega_1(t)e^{i\varphi_1}|e\rangle_1\langle g_0| + \Omega_N(t)e^{i\varphi_N}|e\rangle_N\langle g_0| + H.c., \quad (25)$$

$$H_{ac} = g \left[\sum_{i=1}^{N-1} a_{i,l}|e\rangle_i\langle g_L| + \sum_{i=2}^N a_{i,r}|e\rangle_i\langle g_R| \right] + H.c., \quad (26)$$

$$H_{cf} = v \sum_{i=1}^{(N-1)/2} [b_{2i-1}^\dagger(a_{2i-1,l} + a_{2i,l}) + b_{2i}^\dagger(a_{2i,r} + a_{2i+1,r})] + H.c.. \quad (27)$$

Suppose that initially the atoms are prepared in the separable state $|g_0, g_L, g_R, g_L, \dots, g_R\rangle_{a_1 a_2 \dots a_N}$ while all the cavities and fibers are empty. Then,

under the constraint $2\Omega_1(t)\Omega_N(t)/\sqrt{\Omega_1^2(t) + \Omega_N^2(t)} \ll g$ and the adiabatic following condition, the atoms can eventually appear in the entangled state $|ghz\rangle_{a_1 a_2 \dots a_N} = \cos \alpha |g_0, g_L, g_R, g_L, \dots, g_R\rangle + e^{i(\varphi_1 + \varphi_N)} \sin \alpha |g_L, g_R, g_L, g_R, \dots, g_0\rangle_{a_1 a_2 \dots a_N}$, if $\lim_{t \rightarrow -\infty} \Omega_N(t) > \lim_{t \rightarrow -\infty} \Omega_1(t) = 0$, $\lim_{t \rightarrow \infty} \Omega_{1,N}(t) = 0$ and $\lim_{t \rightarrow \infty} [\Omega_1(t)/\Omega_N(t)] = \tan \alpha$.

III. REALIZATION AND DISCUSSION

A possible implementation of our scheme for $N = 3$ can be realized by using driving pulses with the Rabi frequencies of the shapes [40]

$$\Omega_1(t) = \Omega_0 \sin \alpha e^{-(t-\tau)^2/T^2}, \quad (28)$$

and

$$\Omega_3(t) = \Omega_0 e^{-(t+\tau)^2/T^2} + \Omega_0 \cos \alpha e^{-(t-\tau)^2/T^2}. \quad (29)$$

For illustration, we display in Fig. 3 the time-dependence of the pulses $\Omega_{1,3}(t)$, the probabilities $P_n(t) = |\langle \phi_n | \Phi_1(t) \rangle|^2$ of finding the system in the states $|\phi_n\rangle$ ($n = 1, 3, \dots, 11$) and the fidelity $F = |\langle GHZ | \Phi_1(t) \rangle|^2$ with the parameters $\alpha = \pi/4$, $\Omega_0 = 0.1g$, $\tau = 50/g$ and $T = 80/g$. As is clear from the top figure, the pulses (28) and (29) with the above-chosen parameters satisfy the conditions Eq. (20) and Eq. (22) with $X(\infty) = \tan \alpha$. The middle figure shows that P_1 is decreasing from 1 to 0.5, P_{11} is instead increasing from 0 to 0.5, while $P_3 = P_5 = P_7 = P_9$ remains negligible all the time (in fact $P_{3,5,7,9}$ is increasing from 0 until a maximum value of about 0.0032 when $gt \simeq 38$ but then is decreasing back to 0). In theory, the GHZ state is generated asymptotically in the long-time limit. However, as seen from the bottom figure, at $gt = 100$ the fidelity is already $F(gt = 100) \simeq 0.99$ and at $gt = 170$ it is almost unity: $F(gt = 170) \simeq 0.9999$.

Next, we will analyze the robustness of adiabaticity condition against the pulse shapes of classical fields. From Ref. [40], we know that our scheme needs an optimal range of τ related to T to achieve a preferable adiabaticity. So during the evolution, a major challenge is to choose an optimal relation between τ and T . The dependence of the fidelity F on the pulses' parameters shown in Fig. 4 indicates that F is larger than 99% for τ/T within the range $0.37 \leq \tau/T \leq 1.11$. This means that our scheme could content with the preferable adiabaticity in a relatively large range.

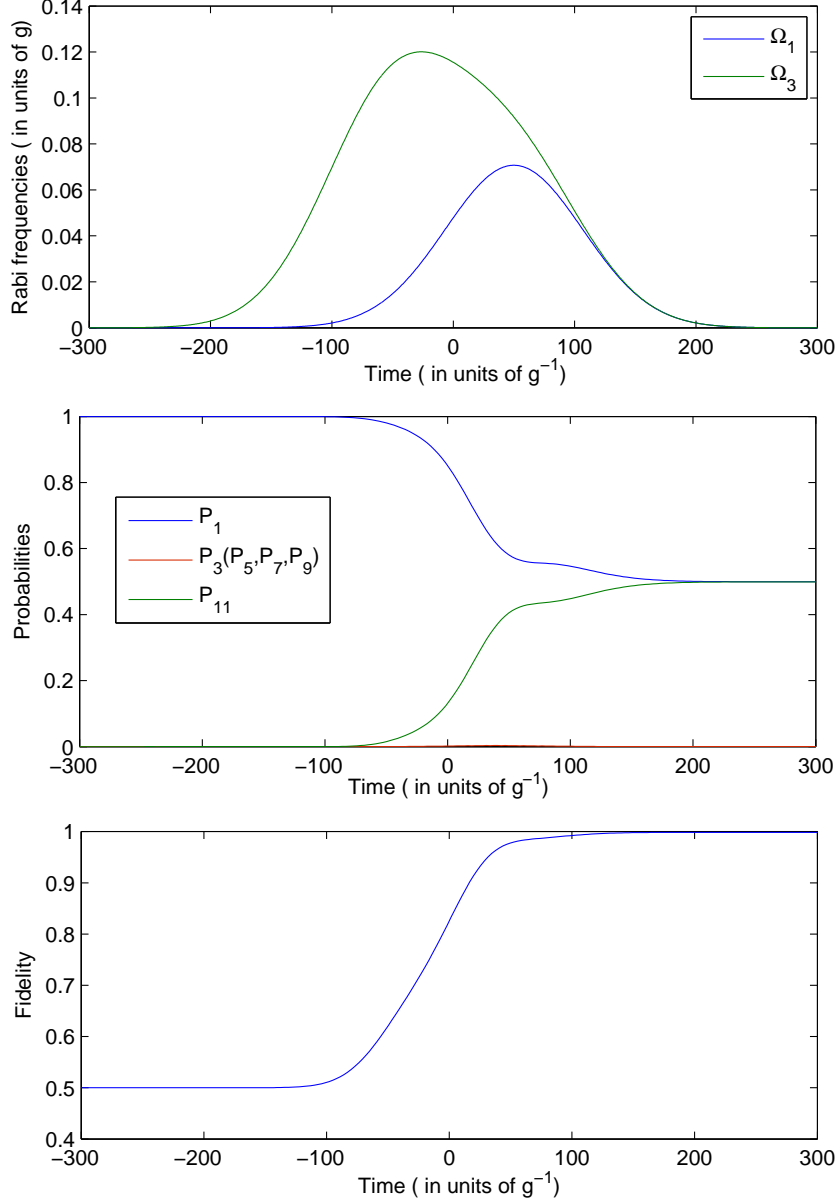


FIG. 3: The time dependence of (top) the Rabi frequencies of the driving lasers $\Omega_1(t)$ and $\Omega_3(t)$, (middle) the probabilities $P_1, P_3, P_5, P_7, P_9, P_{11}$ of finding the system in states $|\phi_1\rangle, |\phi_3\rangle, |\phi_5\rangle, |\phi_7\rangle, |\phi_9\rangle, |\phi_{11}\rangle$, respectively, and (bottom) the fidelity F . The parameters used are $\alpha = \pi/4$, $\Omega_0/g = 0.1$, $g\tau = 50$ and $gT = 80$.

So far the whole system is treated as absolutely isolated from the environment, i.e., we have totally omitted the decoherence effect in our system. In order to confirm the validity of our scheme, we now discuss on the influence of decoherence induced by cavity decay, fiber decay and atomic spontaneous emission. To account for the decoherence we resort to the

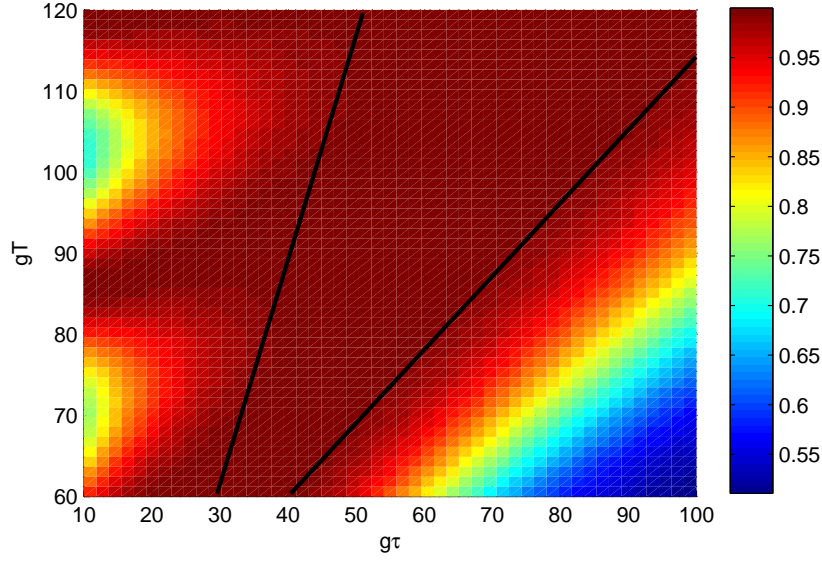


FIG. 4: Density plot of the fidelity F at $gt = 300$ as a function of $g\tau$ and gT for $\Omega_0/g = 0.1$ and $v/g = 10$. The two straight lines indicate the boundaries of the region within which the fidelity is almost one.

master equation for the density matrix $\rho(t)$ of the whole system which has the standard form

$$\begin{aligned}
 \dot{\rho} = & -i[H, \rho] - \sum_{f=1}^2 \frac{k_f}{2} \left(b_f^\dagger b_f \rho - 2b_f \rho b_f^\dagger + \rho b_f^\dagger b_f \right) \\
 & - \left[\sum_{i=1}^2 \frac{\kappa_i}{2} \left(a_{i,l}^\dagger a_{i,l} \rho - 2a_{i,l} \rho a_{i,l}^\dagger + \rho a_{i,l}^\dagger a_{i,l} \right) \right. \\
 & \left. + \sum_{i=2}^3 \frac{\kappa_i}{2} \left(a_{i,r}^\dagger a_{i,r} \rho - 2a_{i,r} \rho a_{i,r}^\dagger + \rho a_{i,r}^\dagger a_{i,r} \right) \right] \\
 & - \sum_{i=1}^3 \sum_{j=g_0, g_L, g_R} \frac{\gamma_{ij}}{2} \left(S_{ij}^\dagger S_{ij}^- \rho - 2S_{ij}^- \rho S_{ij}^\dagger + \rho S_{ij}^\dagger S_{ij}^- \right), \quad (30)
 \end{aligned}$$

where $S_{ij}^\dagger = |e\rangle_i \langle j|$, $S_{ij}^- = |j\rangle_i \langle e|$, k_f (κ_i) denotes the decay rate of the fibers (cavities) and γ_{ij} is the spontaneous emission rate of the atoms. We assume $k_f = k$, $\kappa_i = \kappa$ and $\gamma_{ij} = \gamma = \gamma_0/3$ for simplicity. The fidelity F versus the ratios Ω_0/g and κ/g (Ω_0/g and γ/g) is displayed in the top (bottom) panel of Fig. 5. We can see from Fig. 5 that with the increasing of the laser intensity, the decoherence caused by the atomic spontaneous emission is getting smaller and smaller, while the decoherence caused by the cavity decay is becoming greater and greater. The reason is that the adiabatic passage is just likely to evolve within

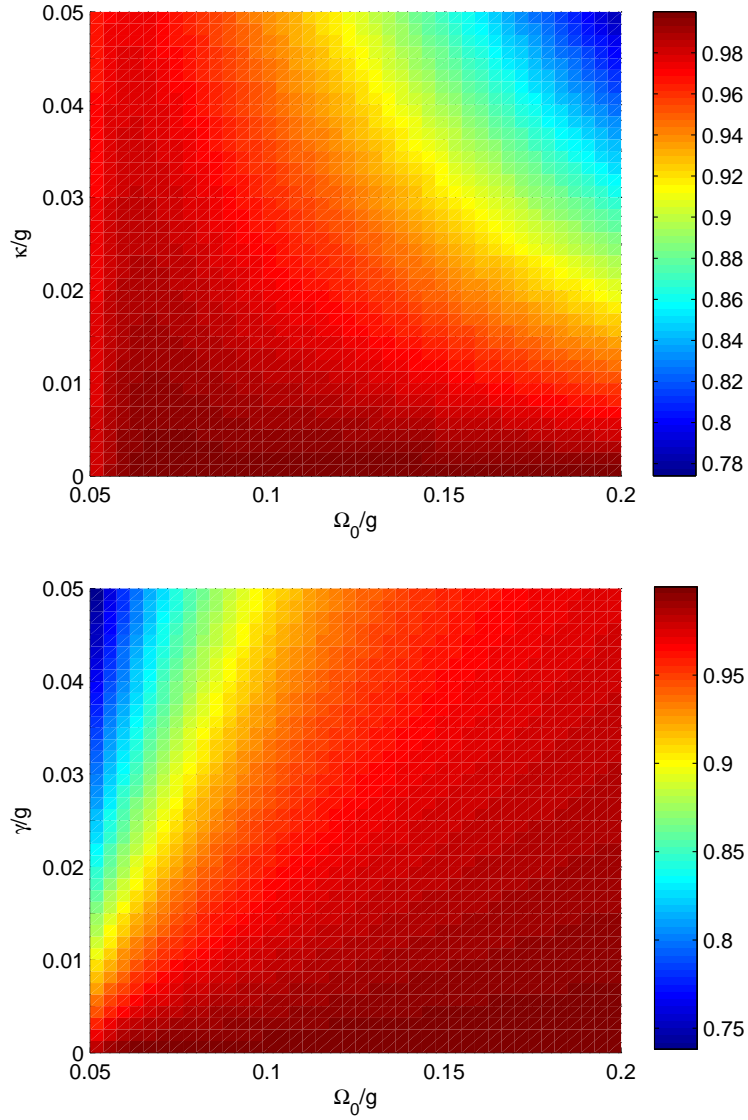


FIG. 5: Density plot of the fidelity F at $gt = 300$ as a function of (top) Ω_0/g and κ/g and (bottom) Ω_0/g and γ/g .

the dark state subspace under relatively large laser intensity, and to violate the influence caused by the spontaneous emission as well. However, the probabilities that the cavity fields are excited increase with laser intensity, which in turn increase the dissipation caused by the cavity decay finally. An appropriate value Ω_0 should be chosen by taking into account both the factors (spontaneous emission of atoms and decay of cavities) as the two error sources cannot be avoided simultaneously. The fidelity F versus the cavity decay κ/g and the fiber decay k/g is shown in Fig. 6, where we neglect the spontaneous emission of atoms. As

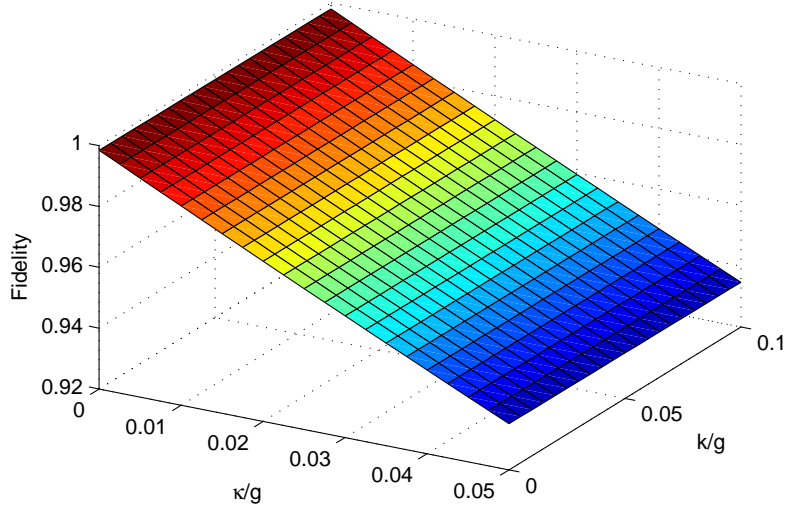


FIG. 6: The fidelity F as a function of κ/g and k/g for $\Omega_0/g = 0.1$, $v/g = 10$ and $gt = 300$.

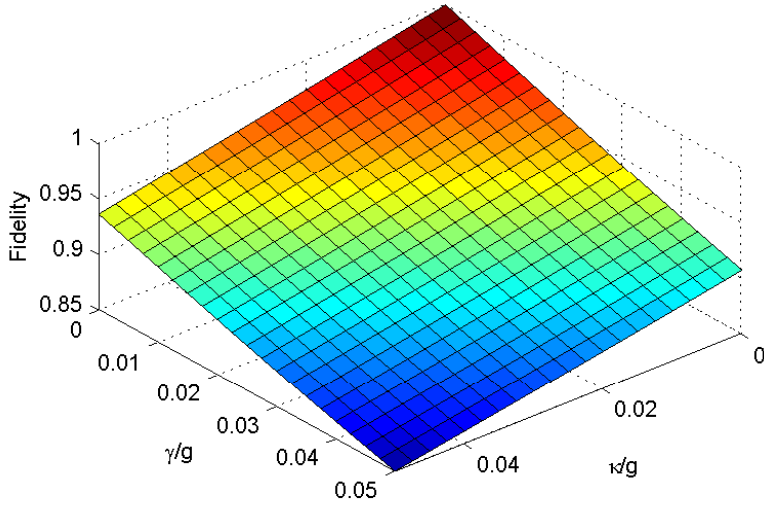


FIG. 7: The fidelity F as a function of κ/g and γ/g for $\Omega_0/g = 0.1$, $v/g = 10$ and $gt = 300$.

seen from the figure, the fidelity F decreases with the increasing cavity decay, but is almost unaffected by the fiber decay. Even though we set $k/g = 0.1$ (and $\gamma/g = \kappa/g = 0$) the fidelity is still as high as $F = 0.998$, so the decoherence due to the fibers hardly influences the quality of the generated state. The fidelity F versus the decay of cavities κ/g and the spontaneous emission of atoms γ/g is shown in Fig. 7 with the fiber decay ignored. We see from Fig. 7 that F decreases with the increasing of both the cavity decay and the atomic

spontaneous emission. For a relative large value of $\gamma/g = \kappa/g = 0.05$, the fidelity is still about $F = 0.850$ when the laser intensity $\Omega_0/g = 0.1$ chosen in our scheme. Therefore, our scheme is robust in realistic conditions.

Finally, let us discuss on the experimental feasibility. The parameters $g = 2\pi \times 75$ MHz, $\gamma = 2\pi \times 2.62$ MHz and $\kappa = 2\pi \times 3.5$ MHz are achievable in optical cavities with the wavelength in the region 630 – 850 nm in recent experiments [45, 46]. A near-perfect fiber-cavity coupling with an efficiency larger than 97.20% can be realized using fiber-taper coupling to high-Q silica microspheres [47]. The optical fiber decay at a 852 nm wavelength is about 2.2 dB/km [48, 49], which corresponds to the fiber decay rate of $k = 0.152$ MHz, which is lower than the cavity decay rate. With these parameters, we will obtain a high fidelity F , meaning that it is possible to realize our scheme in a realistic experiment.

IV. CONCLUSION

In conclusion, we have proposed a one-step scheme for deterministic generation of GHZ states for any number of atoms individually trapped in spatially separated cavities connected with short optical fibers via adiabatic passage by appropriately tailoring the external driving laser fields. The figure of merit is that we need not to precisely control the generation time: the desired state emerges as a steady state of the system. It is interesting that the generation time (i.e., the time it takes to generate the GHZ state with a desired quality, i.e., $F = 1 - \varepsilon$ with a predetermined small ε) does not increase with the number of atoms. This is of importance from the view point of decoherence when dealing with a large-sized system such as entanglement of a big number of atoms. We have also numerically calculated the influences of the driving laser's parameters as well as of the decoherence effect caused by the atom spontaneous emission and the cavity/fiber decay on the quality of the generated state in terms of fidelity. The numerical results have revealed that a relatively high fidelity of three-atom GHZ states can be obtained in the presence of the above-mentioned influencing factors. Therefore, we do hope that within the current experimental technology it would be possible to realize our scheme.

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